

Phase coherence appearance in thin superconducting film with strong disorder. The return to the Mendelssohn model.

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Abstract

It is shown that transition from the mixed state without the phase coherence into the mixed state with the long-range phase coherence (i.e. into the Abrikosov state) of superconductors without disorder must be first order phase transition. The phase coherence appearance in thin superconducting film with strong disorder is considered. The observed smooth transition is explained by increasing of the effective fluctuation dimensionality (from zero to one) in superconductors with strong disorder. Mendelssohn model is used for the explanation of the resistive properties of films with strong disorder.

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1 Introduction

Investigation of the fluctuations have change the habitual notion about nature of the mixed state of type II superconductors. The investigation of conventional superconductors has shown already that the habitual notion based on the results of the mean field approximation is no quite right. But results of this investigation remain no enough widely known for the present. Therefore the most scientists connect the change of the habitual notion with the fluctuation investigation in high-Tc superconductors (HTSC) [1].

Now there is no reason to think that the fluctuation effects in HTSC differ qualitatively from the one in conventional superconductors. Moreover the comparison [2] shows that many results obtained at the HTSC investigation repeat the one obtained at the investigation of conventional superconductors. But these results were interpreted by different way at HTSC and conventional superconductors investigation.

A sharp change of the resistive properties is observed below the second critical field H_{c2} both in bulk conventional superconductors [3] and in $YBa_2Cu_3O_{7-x}$ single crystals [4] with weak disorder. This sharp change was interpreted in [3] as a transition from the mixed state without phase coherence (called as "one-dimensional" state in this paper) into the Abrikosov state. According to this interpretation the Abrikosov state is the mixed state with long-rang phase coherence. The Abrikosov vortexes are singularity in the mixed state with long-rang phase coherence. The Abrikosov vortexes appear in type II superconductor because a magnetic flux can not be inside a superconducting region with phase

coherence and without singularity [5] (the Meissner effect). Consequently, the existence of the Abrikosov vortexes is evidence of the long-rang phase coherence.

According to the mean field approximation [6] the long-rang phase coherence appears at H_{c2} simultaneously with non-zero superconducting electron density $n_s = |\Psi|^2$. And because the magnetic field can not be inside a superconducting region with long-rang phase coherence and without singularity [5] the Abrikosov vortexes appear at H_{c2} also. This transition into the Abrikosov state was considered as second order phase transition [7]. This opinion can be right in the mean field approximation but it can not be right in the fluctuation theory, because the effective dimensionality of the fluctuation decreases on two near H_{c2} (in the lowest Landau level (LLL) approximation region) [8].

According to the mean field approximation the mixed state with long-rang phase coherence (i.e. the Abrikosov state) can exist only. But according to the fluctuation theory the mixed state without long-rang phase coherence can exist also. According to the fluctuation theory the density of the superconducting electrons above H_{c2} is not equal zero and the length of the phase coherence is small. Across a magnetic field direction it is equal approximately $(\Phi_0/H)^{0.5}$. Where Φ_0 is the flux quanta, H is the magnetic field value. This state I call as the mixed state without the phase coherence. The length of the phase coherence in the mixed state without the long-rang phase coherence can be defined from the correlation function $g(R) = \langle \Psi^*(R')\Psi(R' + R) \rangle$.

The correlation function of three-dimensional superconductors in a high magnetic field (in the LLL region) in the linear approximation is [9]

$$g(R) = g(r, z) = A_H \exp(-|z|/\xi_z) \exp(-r^2/\xi_r^2) \quad (1)$$

at $|z| \gg \xi_z$ and $r \gg \xi_r$. Where A_H is a coefficient; $r = (x^2 + y^2)^{0.5}$; $\xi_z = (\Phi_0/2\pi(H - H_{c2}))^{0.5}$; $\xi_r = (2\Phi_0/\pi H)^{0.5}$. A magnetic field is parallel to z . ξ_z and ξ_r are the length on which the phase coherence is exist (i.e. ξ_z and ξ_r are the coherence length). The relation for the longitudinal coherence length $\xi_z = (\Phi_0/2\pi(H - H_{c2}))^{0.5}$ can be valid in the linear approximation only and must be renormalized near H_{c2} . Whereas the transversal coherence length $\xi_r = (2\Phi_0/\pi H)^{0.5}$ changes little near H_{c2} and ξ_r value is close to $(\Phi_0/H)^{0.5}$ in the whole LLL region. This means that the correlation function of bulk superconductor in the LLL region is similar to the one of one-dimensional superconductor. The correlation function at H_{c2} differs qualitatively from the one in zero magnetic field. All components of the coherence length, i.e. ξ_x , ξ_y and ξ_z , increase up to the infinity at the critical temperature where the second order phase transition takes place.

Thus, if we define the phase coherence by the correlation function we can conclude that the long-rang phase coherence can not be in the mixed state of type II superconductors. On other hand we know that the Abrikosov state is the mixed state with long-rang phase coherence. Consequently, the definition of the phase coherence by the correlation function is unsuited for the mixed state.

It must follow from a right definition that the existence of the singularities (the Abrikosov vortexes) is evidence of the phase coherence.

The Meissner effect is the consequence of the relation [10]

$$\frac{\Phi_0}{2\pi} \int_l dR \frac{d\phi}{dR} = \int_l dR \lambda_L^2 j_s + \Phi \quad (2)$$

where l is a closed path of integration; $\lambda_L = (mc/e^2 n_s)^{0.5}$ is the London penetration depth; j_s is the superconducting current density; Φ is the magnetic flux contained within the closed path of integration l . If the singularity is absent $\int_l dR d\phi/dR = 0$. In this case the relation (2) is the equation postulated by F. and H. London [11] for the explanation of the Meissner effect [12] (see [10]). In the Abrikosov state $(\int_l dR d\phi/dR)/2\pi = n$ is a number of the Abrikosov vortex contained within the closed path of integration. The relation (2) is gauge-invariant. It is valid if the phase coherence exists along the closed path of integration. Consequently, we can use the relation (2) for the definition of the phase coherence: the phase coherence exists in some region if the relation (2) is valid for any closed path in this region. It is obvious that according to this definition the long-rang phase coherence exists both in the Meissner and Abrikosov states. If the closed path is great (i.e. if its radius $\gg \lambda_L$) then $\int_l dR \lambda_L^2 j_s$ value is small in comparison with other terms of (2). In this case the condition of the phase coherence is $(\int_l dR d\phi/dR)/2\pi = \Phi/\Phi_0 = n$.

In the mixed state without the phase coherence the relation (2) is valid for the closed path of integration l with a radius $r = l/2\pi < (\Phi_0/H)^{0.5}$ only (across magnetic field). Therefore, in this case the magnetic field can penetrate in a superconductor without singularities (i.e. without Abrikosov vortexes) as well as in the normal state. Because in the mixed state without the phase coherence the transversal length of the phase coherence $\xi_r \simeq (\Phi_0/H)^{0.5}$ changes little near H_{c2} two characteristic lengths only $((\Phi_0/H)^{0.5}$ and sample size L) are across magnetic field direction in a superconductor without disorder. Therefore the long-rang phase coherence appearance must be a first order phase transition in ideal superconductors. The length of the phase coherence must change by jump from $(\Phi_0/H)^{0.5}$ to L at this transition.

The resistivity has different nature in the mixed states with and without the phase coherence. The cause of the resistivity in the mixed state without the phase coherence does not differ qualitatively from the one in the normal state. The resistivity value is decreased in the consequence of the paraconductivity [3]. Whereas, the voltage can be in the Abrikosov state only if the phase difference changes in time. The change of the phase difference in time in the Abrikosov state means the vortex flow. The resistivity caused by the vortex flow is called "flux flow resistivity" [10]. But it is no quite right denomination because the magnetic flux does not flow. I will use more right denomination "vortex flow resistivity" instead of "flux flow resistivity". I will considered the resistivity in the mixed state without the phase coherence as the paraconductivity regime and in the Abrikosov state as the vortex flow regime.

The vortex flow is impeded by disorder of the superconductor. In the consequence of this the current-voltage characteristics of superconductor in the Abrikosov state can be non-Ohmic. And in some cases the resistivity can be equal zero below the critical current [10]. This phenomena is called as vortex pinning effect [13]. It is obvious that the vortex pinning can not be in the state without the phase coherence because the vortex (as the singularity in the state with the phase coherence) can not exist without the phase coherence. Therefore the vortex pinning effect may be considered as the consequence of the phase coherence.

Consequently, a change of the resistive properties must be observed first of all at the long-rang phase coherence appearance. The vortex pinning appears and a transition from the paraconductivity regime to the vortex flow regime takes place there. This change must be sharp in the ideal superconductor. Therefore it is obvious that the sharp change of the resistive properties observed in bulk superconductors with weak disorder [3] at $H_{c4} < H_{c2}$ is the transition into the Abrikosov state, because the vortex pinning appears and the vortex flow resistivity decreases sharply at H_{c4} . (The designation H_{c4} for the position of the transition into the Abrikosov state was proposed in my paper [14]). The observation of the sharp change in [3] means that the transition into the Abrikosov state in bulk superconductors with weak disorder can be close to the one in the ideal superconductor. The length of the phase coherence changes abruptly from $(\Phi_0/H)^{0.5} = 10^{-5} - 10^{-6} cm$ to a sample size $L = 0.01 - 1 cm$ in real cases.

The same transition is observed in bulk conventional superconductor [3] and in $YBa_2Cu_3O_{7-x}$ [4] (it was shown in [2] for example). But a conception of the vortex lattice melting has become very popular at the HTSC investigation [15]. Therefore the sharp change of the resistive properties (kink) in $YBa_2Cu_3O_{7-x}$ was interpreted in [4] and other papers as the vortex lattice melting. This interpretation is very popular, but it can not be right.

According to this interpretation the transition from the vortex lattice state into the vortex liquid state is observed at H_{c4} . (Because the same transition is observed in [3] and in [4] I use the designation H_{c4} in both cases). But the vortex liquid is the mixed state with long-rang phase coherence (i.e. the Abrikosov state), because the existence of the vortexes is evidence of the long-rang phase coherence. According to the right definition by the relation (2) the existence of the long-rang phase coherence does not depend on the crystalline long-rang order of the vortex. Consequently, the first order phase transition from the vortex liquid into the mixed state without the phase coherence must be observed above the vortex lattice melting in ideal superconductor. The sharp change of the resistive properties must be observed at this transition. But no sharp change is observed above H_{c4} both in conventional superconductors [3] and in $YBa_2Cu_3O_{7-x}$ [4]. Consequently, the transition from the mixed state without the phase coherence into the Abrikosov state is observed both in [3] and in [4]. Result of [16] shows that this transition in bulk superconductors with weak disorder can be first order indeed.

It is proposed in vortex lattice melting theories that the Abrikosov state is the vortex lattice. This opinion is based on the Abrikosov solution [6]. But it not proved that this solution is valid in thermodynamic limit. It must be emphasized that the direct observations ([17] and many others) do not prove the validity of the Abrikosov solution [6]. The experimental results are obtained by the investigation of real samples whereas the Abrikosov solution is obtained for the ideal case. Larkin [18] has shown that the crystalline long-rang order of the vortex lattice is unstable against the introduction of random pinning. Consequently, the crystalline long-rang order does not exist in real superconductors and can be in the ideal case only. But the existence of the Abrikosov state in the ideal case has not been proved for the present because the mean field approximation is not valid in this case.

Therefore, the vortex lattice melting theories are unsatisfactory in principle, since they start from the state in which the translation symmetry has been broken by hand [19]. Some theorists consider no the vortex lattice melting but the solidification transition of vortexes [20, 21]. They do not propose the Abrikosov state existence, but try to find the transition to it. But the phase coherence is defined by the correlation function in the solidification theory. According to this definition the long-rang phase coherence can not exist without the crystalline long-rang order of the vortexes. It is claimed in some works [22] that the phase coherence can remain short-ranged even in the vortex solid phase. Therefore the solidification theories [20, 21] find the transition into the Abrikosov state as the vortex lattice but no as the mixed state with long-rang phase coherence. Most authors find this transition [20]. And few authors [21] only state that this transition is absent. I agree with few authors [21]. But the absence of the solidification transition does not mean that the transition into the Abrikosov state is absent, because the definition of the phase coherence by the correlation function is unsuited for the mixed state. According to the right definition of the phase coherence the Abrikosov state is the mixed state with long-rang phase coherence but no the vortex lattice.

Thus, the Abrikosov solution [6] gives qualitatively incorrect result because the mean field approximation is not valid for description of the mixed state of type II superconductors. The transition in the Abrikosov state in ideal superconductor can be first order only and can not be second order. The sharp change of the resistive properties must be observed at this transition. But this sharp change is observed in few bulk samples with weak disorder [3, 4] only. The kink in $YBa_2Cu_3O_{7-x}$ was observed first in 1990 year [23] only when enough quality single crystals were obtained. No sharp change is observed both in thin films with weak disorder [24] and in all samples with strong disorder. The transition into the Abrikosov state in bulk superconductors becomes smooth with increasing of disorder amount [25].

The absence of any features of the resistive properties in amorphous $Nb_{1-x}O_x$ films was interpreted in our paper [24] as the absence of the transition into the Abrikosov state down to very low fields. But Theunissen and Kes [26] contend

that we do not observe any feature because our measuring current is extremely high in comparison to the critical current. I agree with the Theunissen and Kes that the vortex pinning appearance can not be observed if the measuring current is extremely high in comparison to the critical current. But I contend that the transition from the paraconducting regime to the vortex flow regime can not be without features in the ideal sample.

The fluctuation decreases the vortex flow resistivity as well as the resistivity above the transition into the Abrikosov state [27]. The fluctuation value increases near the transition. Therefore features ought be expected at the transition from the paraconducting regime to the vortex flow regime. Sharp feature is observed in enough homogeneous bulk superconductors [28]. At the transition into the Abrikosov state not only the vortex pinning appears but also the vortex flow resistivity decreases sharply. Below the transition the vortex flow resistivity dependence has a minimum [28]. Such dependence differ qualitatively from the mean-field vortex flow resistivity dependence [29] and is explained in [28] by fluctuation influence.

The authors of the paper [26] state that these features of the vortex flow dependencies coincide with the peak effect in the critical current. But this is right no always. Our investigations [30] have shown that these features are observed in all enough homogeneous bulk samples both with and without the peak effect. And only in no enough homogeneous samples the "classical" flux flow resistivity dependencies [31] are observed. Therefore these features ought be considered as universal for homogeneous bulk superconductors.

The features of the vortex flow resistivity are observed in paper [26] (see the inset of Fig.7). These features are observed below the field value where the vortex pinning appears. As it was shown above the vortex pinning is the consequence of the phase coherence. Consequently the features of the vortex flow resistivity are observed below the phase coherence appearance in thin films [26] as well as in bulk superconductors [28]. But these features are no sharp in thin films [26].

The comparison of the pinning values (made in [26]) shows that the amount of disorder in our amorphous $Nb_{1-x}O_x$ films [24] is smaller than in the a-NbGe films used in [26]. Therefore the features at the transition from the paraconducting regime to the vortex flow regime must be more sharp in our films. The features of the vortex flow resistivity can be observed at high measuring current. But no features were observed on the resistivity dependencies of the amorphous $Nb_{1-x}O_x$ films in our paper [24]. Therefore I think that the phase coherence appearance in thin film is not universal, but that it depends on the amount of disorder: the phase coherence appears in a middle field in the a-NbGe films with an intermediate strength of disorder [26] and does not appear down to very low fields in the $Nb_{1-x}O_x$ films with extremely small pinning [24].

This claim is confirmed also by our theoretical results [32]. The transition into the Abrikosov state is connected with the Abrikosov parameter $\beta_a = \overline{n_s^2}/\overline{n_s}^2$. Where $\overline{n_s^2} = (\int_V dR n_s^2)/V$ and $\overline{n_s} = (\int_V dR n_s)/V$ is the spa-

tial average value of the superconducting electron density. Kleiner, Roth and Author [33] have shown that the minimum possible value β_a is equal $\beta_A \simeq 1.16$ if $n_s(R)$ are lowest-Landau-level (LLL) functions. The minimum β_a value only is considered in the mean field approximation. Therefore the triangular vortex lattice appears below H_{c2} according to this approximation because this state corresponds the minimum β_a value [10]. But the exact thermodynamic average $\langle \beta_a \rangle$ is not equal β_A in the consequence of the fluctuation. The $\langle \beta_a \rangle$ value changes from 2 above the H_{c2} critical region to a value closed to β_A below the H_{c2} critical region [32]. The $\langle \beta_a \rangle$ dependence of two-dimensional superconductor was calculated in work [34] for whole LLL region. $\langle \beta_a \rangle$ is not equal β_A at a finite temperature. Therefore the transition into the Abrikosov state can be when the $\langle \beta_a \rangle$ value is smaller then a critical value, i.e. when $\langle \beta_a \rangle > -\beta_A < (\beta_a - \beta_A)_c$. According to [32] the $(\beta_a - \beta_A)_c$ value of two-dimensional ideal superconductor decreases with sample size increasing. In real case the sample size ought to be replaced by a effective distance between disorders. The $\langle \beta_a \rangle > -\beta_A$ value decreases with the magnetic field (or temperature) decreasing [34, 32]. Therefore according to [32] the position of the transition into the Abrikosov state depends on the amount of disorder in thin films.

Thus, the absence of any sharp change of the resistive properties in thin films with weak disorder [24] can be explained by the absence of the transition into the Abrikosov state down to very low fields. The absence of the long-range phase coherence below H_{c2} in these films is confirmed by a recent investigation of the nonlocal resistivity [5]. But it is obvious that the long-range phase coherence is appeared in superconductors with strong [25] and intermediate [26] strength of disorder, because the pinning effect is observed in these cases. Consequently, the phase coherence appearance in superconductors with strong and intermediate strength of disorder differs from the ideal case. The transition into the Abrikosov state is smooth in these superconductors. The length of the phase coherence does not change by jump but increases gradually with the magnetic field (or the temperature) decreasing.

In the present work the phase coherence appearance in thin films with strong disorder is considered. The resistive properties of the thin $Nb_{1-x}O_x$ films with weak and strong disorder is compared. In order to explain the difference of the phase coherence appearance in superconductors with strong disorder from the ideal case it is proposed to return to the Mendelssohn model.

2 SAMPLE PREPARATION AND CHARACTERISTICS

The $Nb_{1-x}O_x$ films were produced by magnetron sputtering of Nb in an atmosphere of argon and oxygen. Changing the oxygen we produced the films with different oxygen contents. The transmission electron microscopy investiga-

tion shown that the films with small oxygen contents have small grain structure whereas the films with greater oxygen contents are amorphous. The temperature of the superconducting transition, T_c , of the films decreases with the oxygen content increasing. At the oxygen content $x > 0.2$ the critical temperature $T_c < 2\text{ K}$. In the present work the amorphous films with $x \simeq 0.2$ and the films with small ($\simeq 10\text{ nm}$) grain structure with $x \simeq 0.08$ were used. The oxygen content was determined by Auger analysis with a relative error 0.3. The critical temperature of used amorphous films $T_c = 1.8 - 3\text{ K}$ and of the used films with small grain structure $T_c = 5.7\text{ K}$. $(dH_{c2}/dT)_{T=T_c} = 22\text{ kOe/K}$ for the amorphous films and $(dH_{c2}/dT)_{T=T_c} = 6\text{ kOe/K}$ for the crystalline films. The temperature dependence of normal resistivity of both films is weak. The normal resistivity $\rho_n = 3 \cdot 10^{-7}\Omega m$ of the films with small grain structure decreasing weakly with temperature decreasing. Whereas the resistivity $\rho_n = 20 \cdot 10^{-7}\Omega m$ of the amorphous films increases with decreasing temperature. This change can be connected with weak localization.

A perpendicular magnetic field up to 50 kOe produced by a superconducting solenoid was measured with relative error 0.0005. The temperature was measured with a relative error 0.001. The resistivity was measured with a relative error 0.0001.

3 RESULTS AND DISCUSSION

The scaling of the resistivity was observed in amorphous $Nb_{1-x}O_x$ [24] and a-NbGe [26] films with weak disorder. This scaling is general consequence of the fluctuation Ginzburg-Landau theory in the lowest Landau level (LLL) approximation [2]. The scaling of various properties is consequence of the scaling of the spatial average value of the superconducting electron density $\overline{n_s}$ [2]. Strictly speaking the scaling of $\overline{n_s}$ is valid in the state without the phase coherence only because the $\overline{n_s}$ value changes at the phase coherence appearance. But this change is very small [16]. Therefore the dependencies of the specific heat [35], magnetization and other thermodynamic properties depended only on the $\overline{n_s}$ value are close to the scaling in the states both with and without the phase coherence.

But the transport properties depend strongly on the phase coherence. Therefore the resistivity dependencies can be close to the scaling in the state without the phase coherence only. The sharp deviation of the conductivity dependencies from the scaling is observed in bulk superconductors at the transition into the Abrikosov state [36, 30]. Consequently, the scaling of the resistivity is evidence of the absence of the phase coherence and the absence of this scaling is evidence of the phase coherence. Therefore we can use the comparison of the resistive dependencies with the scaling for the investigation of the phase coherence appearance.

In contrast of the results [24, 26] the resistivity dependencies of films with

strong disorder clearly deviate from the scaling Fig.1. According to the scaling law the $\Delta\sigma(Gi_{2D}ht)^{0.5}/t$ is a universal function of $(t - t_{c2})/(Gi_{2D}ht)^{0.5}$. Here $\Delta\sigma = \sigma(t, h) - \sigma_n$ is the excess conductivity of two-dimensional superconductor (i.e. thin film); $\sigma_n = 1/\rho_n$ is the normal conductivity of the film; $Gi_{2D} = k_B T_c / H_c^2(0) d\xi^2(0)$ is the Ginzburg number of two-dimensional superconductor; $t = T/T_c$; $h = H/H_{c2}(0)$; $H_{c2}(0) = -T_c(dH_{c2}/dT)_{T=T_c}$; $H_c(0)$ is the thermodynamic critical field at $T = 0$; $\xi(0)$ is the coherence length at $T = 0$; d is the film thickness. In Fig.1 the $(1 + (\Delta\sigma/\sigma_n)(h/t)^{0.5})^{-1}$ versus $(t - t_{c2})/(ht)^{0.5}$ dependencies of the $Nb_{1-x}O_x$ film with small grain structure in different magnetic fields are shown. (The same coordinates were used on Fig.4 of Ref.[24]). The Fig.1 demonstrates clearly that the scaling is not observed in the film with strong disorder.

The excess conductivity of the film with strong disorder deviates clearly from the scaling already above H_{c2} . As demonstrated in Fig.2 the $\Delta\sigma(Gi_{2D}ht)^{0.5}d/t\sigma_0$ versus $(h - h_{c2})/(Gi_{2D}ht)^{0.5}$ dependencies of the $Nb_{1-x}O_x$ film with small grain structure begins to deviate from the one of the amorphous $Nb_{1-x}O_x$ film at $h \simeq 1.05h_{c2}$. Where $\sigma_0 = e^2/\hbar$; $h_{c2} = H_{c2}/H_{c2}(0)$. The h_{c2} value is determined by the comparison of the experimental and theoretical paraconductivity dependencies in the linear approximation region. The reliability of this method has been demonstrated at the investigation of the paraconductivity in bulk superconductors. The distinction of the position of the transition into the Abrikosov state from H_{c2} has been discovered first by the using of this method [28]. Later [37] this result was confirmed by determination of H_{c2} value from magnetization measurement.

The deviation of the conductivity dependence from the scaling in thin films with strong disorder differs from the one in bulk superconductor with weak disorder. The sharp deviation is observed in bulk superconductors and it coincides with the qualitative change of the current-voltage characteristic form [36, 30]. The current-voltage characteristics become non-Ohmic. The deviation in the thin film is smooth and is observed above the magnetic field value where the non-Ohmic current-voltage characteristics are observed. For example at $T = 4.2$ K the non-Ohmic current-voltage characteristics are observed in the $Nb_{1-x}O_x$ film with small grain structure at $H/H_{c2} \simeq 0.7$ ($H_{c2} = 9.0 kOe$) whereas the deviation is observed at $H/H_{c2} \simeq 1.05$ (Fig.2).

At $0.2 < H/H_{c2} < 0.65$ the current-voltage characteristics can be described by the relation

$$E = E_0 \sinh(j/j_0) \quad (3)$$

(see Fig.3). According to this relation at $j \ll j_0$, $E \simeq E_0 j/j_0 = \rho_{TAFF} j$. Where $\rho_{TAFF} = E_0/j_0$ is a thermally activated linear resistivity [38]. The ρ_{TAFF} value decreases strongly with magnetic field decreasing: from $3 \cdot 10^{-9} \Omega m$ at $H = 6$ kOe to $10^{-15} \Omega m$ at $H = 2$ kOe (Fig.3). Expected ρ_{TAFF} values at $H/H_{c2} < 0.2$ and at $H/H_{c2} > 0.65$ can be estimated by extrapolation of the

$E_O(H)$ and $j_0(H)$ dependencies (see Fig.5). According to this extrapolation $\rho_{TAFF} = 1.2 \cdot 10^{-8} \Omega m$ at $H = 7$ kOe is close to resistivity value $1.4 \cdot 10^{-8} \Omega m$ measured at $H = 7$ kOe whereas $\rho_{TAFF} \simeq 10^{-20} \Omega m$ at $H = 1$ kOe is too little for experimental measurement. Therefore I may assume that the relation (3) is valid in more wide region than $0.2 < H/H_{c2} < 0.65$. But it is obvious that the relation (3) can be valid only if $\rho_{TAFF} \ll \rho_{scal}$. Where ρ_{scal} is a resistivity value according to the scaling law (i.e. the resistivity in films with weak disorder [24]). The relation ρ_{TAFF}/ρ_{scal} at $T = 4.2$ K ($H_{c2} = 9.0$ kOe) is equal 0.38; 0.07; 0.02; 0.004; 0.0008; 0.00001 at $H = 8; 7; 6; 5; 4; 2$ kOe. I use the measured linear resistivity at $H = 8$ and 7 kOe as the ρ_{TAFF} value. Consequently the relation (3) can not be valid near $H_{c2} = 9.0$ kOe where $\rho_{TAFF} \simeq \rho_{scal}$.

The smooth deviation of the conductivity dependence from the scaling Figs.1,2 means that the length of the phase coherence increases smoothly. This result differs strongly from the ideal case. As it was shown above the ideal phase coherence appearance must be sharp transition because the fluctuation is zero-dimensional in thin film without disorder placed in high perpendicular magnetic field. The observed difference of the phase coherence appearance from the ideal case can be explained by increasing of the effective dimensionality of the fluctuation (from zero to one in film) in samples with strong disorder.

The limit case of strong disorder is the Mendelssohn's "sponge" [39]. The Mendelssohn's "sponge" is a model proposed more than sixty years ago for explanation of the magnetic properties of some superconductors (now we call these superconductors as type II superconductors). Mendelssohn has assumed that the magnetic field can penetrate inside superconducting region because this region is superconducting sponge. Indeed, the magnetic and resistive properties the superconducting sponge are rather like the one of type II superconductors with strong disorder. The magnetization dependence is irreversible in both cases. The critical current can be great in high magnetic field.

The Abrikosov vortex can exist in the superconducting sponge with the phase coherence as well as in type II superconductor. The phase coherence in the superconducting sponge can be defined by the relation (2). The Abrikosov vortex is defined as a singularity of the wave function. The integral $\int_l dR d\phi/dR$ around this singularity is equal 2π . Let us consider the two-dimensional superconducting sponge with long-range phase coherence placed in a magnetic field (see Fig.4A). The integral $\int_l dR d\phi/dR$ around any cell of the sponge must be equal $2\pi n$. Where n is an integer. If H/Φ_0 is smaller than a cell density then $n = 1$ or 0 in a state with minimum value of the free energy. The free energy value increases at $n > 1$ because the superconducting current value increases in this case in accordance with the relation (2). We can say that a cell contains the Abrikosov vortex if $n = 1$ and the Abrikosov vortex is absent if $n = 0$ (see Fig.4). The free energy has minimum in the state with homogeneous vortex density (Fig.4).

The Abrikosov state in thin film is similar to the two-dimensional Mendelssohn sponge Fig.4. The main difference is that the Abrikosov vortex destroys su-

perconductivity near itself in the Abrikosov model (Fig.4B) whereas in the Mendelssohn model it occupies a nonsuperconducting region (Fig.4A). According to the relation (2), near a vortex $\lambda_L^2 j_s = 2mc v_s / e \simeq \Phi_0 / 2\pi r - Hr/2$. Here r is the distance from the vortex center; v_s is the velocity of the superconducting electrons. I consider the case $\lambda_L \gg \xi > d$. $B(r) \simeq H$ in this case. At a r_c value, v_s is equal the critical velocity [9]. Therefore $n_s = 0$ at $r < r_c$. The calculations show that $r_c \simeq \xi$. Disorders acting as pinning centers have lower superconducting parameters. In the limit case they are nonsuperconducting inclusions (Fig.4B). The Mendelssohn sponge (Fig.4A) is a superconductor with nonsuperconducting inclusions size of which b larger than the correlation length, $b > \xi$. Thus, we may consider real superconductors with disorder as intermediate cases between the Mendelssohn's [39] and Abrikosov's [6] models.

Because the Abrikosov vortex destroys superconductivity near itself at $r < r_c \simeq \xi(T)$ and the distance between the Abrikosov vortexes is equal approximately $(\Phi_0/H)^{0.5}$ we may consider the thin film with strong disorder as two-dimensional Mendelssohn sponge with variable width of superconducting threads $w(T, H) \simeq (\Phi_0/H)^{0.5} - \xi(T)$. The $w(T, H)$ value increases with the T and H decreasing. The above relation can be valid at enough great w value only. Therefore this relation can not be used for the evaluation a critical field $H_{c,sp}$ at which $w = 0$. One ought expected that $H_{c,sp} \simeq H_{c2}$ in film with strong disorder. The mean field critical field of the superconducting sponge with cell size $\gg \xi$ and $w < \xi$ can be appreciably higher than H_{c2} . This critical field, as well as the critical field of thin film in parallel magnetic field [9], is equal $H_{c,sp} = 3^{0.5} \Phi_0 / \pi \xi w = 3^{0.5} 2 H_{c2} \xi / w$.

I can not describe qualitatively the resistive dependence of the Mendelssohn sponge with variable $w(T, H)$ value. But it is obvious that this dependence is smooth because the Mendelssohn sponge is a one-dimensional system. In one-dimensional superconductor [40] the length of the phase coherence increases smoothly with temperature decreasing below T_c . In consequence of this the resistive dependence is smooth also. For example see the resistive transition of bulk superconductor in parallel magnetic field in the paper [41]. It is obvious that the $\rho_{TAFF} / \rho_{scal}$ value decreases in the consequence of the increasing of the length of the phase coherence. When the length of the phase coherence has increased up to sample size the ρ_{TAFF} value becomes $\ll \rho_{scal}$. Consequently the length of the phase coherence of the $Nb_{1-x}O_x$ film with small grain structure at $T = 4.2$ K increases from $\simeq (\Phi_0/H)^{0.5}$ at $H = 1.05 H_{c2} = 9.5 kOe$ (Fig.2) to sample size at $H = 6 - 7$ kOe (see above).

A crossover to the vortex creep regime takes place in low magnetic field where $\rho_{TAFF} \ll \rho_{scal}$ (Fig.3). Below I compare the experimental data (Fig.3) with theoretical results obtained in the Kim-Anderson model [42] and obtained in a model of the vortex creep in the Mendelssohn sponge. The Kim-Anderson model [42] describes the vortex creep in type II superconductors with pinning. According to [42] the current-voltage characteristics in the vortex creep regime are described by the relation (3) in which $j_0 = k_B T / B V_j l_j$ (see [38]). Here V_j

is jumping volume and l_j is the jump width. Because the film is thin $V_j = dS_j$, where S_j is the jumping area. We can estimate the $S_j l_j = k_B T / j_0 H d$ value from the experimental dependencies shown partly on Fig.3. The $(S_j l_j)^{1/3}$ value is plotted versus the $(\Phi_0/H)^{1/2}$ value in Fig.5A. The distance between the Abrikosov vortex in the triangular lattice $(2\Phi_0/3^{1/2}H)^{1/2}$ is shown also in Fig.5A. According to Fig.5A the $(S_j l_j)^{1/3}$ value does not exceed $(2\Phi_0/3^{1/2}H)^{1/2}$ value. Therefore I can conclude that the individual vortex creep takes place in our film.

The vortex creep can be in the Mendelssohn sponge if the vortex can jump from one to the other cell (see Fig.4A). A vortex can jump to next cell of the Mendelssohn sponge if superconductivity has been destroyed in the cell wall. The energy must increase at this on $\xi(T)dw f_{GL}$, where f_{GL} is the difference of the free-energy density in the normal and the superconducting phase [9]. Consequently the probability of the jump is proportional to $\exp(\xi(T)dw f_{GL}/k_B T)$. According to the Ginzburg-Landau theory $f_{GL} = (H_c^2(T)/8\pi)(1 - (\hbar m v_s \xi)^2)$ [9]. Because the magnetic flux inside the cell is smaller than Φ_0 a superconducting current is around the cell (see relation (2)). Therefore the transport current increases the velocity of the superconducting electrons in one side of the cell and decreases in opposite side. Consequently the transport current increases the rate of thermally activated jumps of vortexes in one side and decreases it in opposite side (Fig.4A). The transport current change weakly the v_s value. Therefore we can use following relation for the vortex velocity

$$v_{vor} = 2l_j \omega_0 \exp\left(\frac{\xi dw f_{GL}}{k_B T}\right) \sinh\left(\frac{\xi dw v_{tr}}{k_B T} \frac{df_{GL}}{dv_s}\right) \quad (4)$$

where ω_0 is an attempt frequency; v_{tr} is a change of the v_s value in consequence of the transport current. Because the voltage is proportional the vortex velocity the current-voltage characteristics of the Mendelssohn sponge in the creep regime conforms to the relation (3). $\xi w dv_{tr}(df_{GL}/dv_s) = (\xi d\Phi_0/8\pi^2)(1 - (\xi/b)^2(1 - HS_{cell}/\Phi_0)^2)(1 - HS_{cell}/\Phi_0)j \simeq (\xi d\Phi_0/8\pi^2)j$, where S_{cell} is cell area; j is the transport current density. Consequently, the j_0 value (see the relation (3)) of the two-dimensional Mendelssohn sponge is equal $j_0 \simeq 8\pi^2 k_B T / \xi d \Phi_0$. For the parameter values of our film ($\xi \simeq 20nm$, $d = 20nm$), $j_0 \simeq 5 \cdot 10^9 A/m^2$ at $T = 4.2 K$. The experimental j_0 values (Fig.3) are smaller approximately by a factor of 50 - 100.

The observed decreasing of the $E_0 = H l_j \omega_0 \exp(\xi dw f_{GL}/k_B T)$ value with magnetic field decreasing (see Fig.3) can be explained qualitatively by the increasing of the w value. The experimental dependence $\ln(E_0/E_{0,Hc2})$ versus $(H_{c2}/H)^{0.5} - 1$ is plotted in Fig.5B. Where $E_{0,Hc2}$ is a normalizing voltage value. I following a model according to which $w(H) \simeq \xi((H_{c2}/H)^{0.5} - 1)$. The dependence shown on Fig.5B corresponds qualitatively to this model. $E_{0,Hc2} = 2H l_j \omega_0$ in this case. The slope of the dependence $\ln(E_0/E_{0,Hc2})$ versus $(H_{c2}/H)^{0.5} - 1$ is equal $-\xi^2 df_{GL}/k_B T$ in this model. According to Fig.5B $\xi^2 df_{GL}/k_B T \simeq 16$ whereas the $\xi^2 dH_c^2/8\pi$ value of our $Nb_{1-x}O_x$ film at $T = 4.2$

K is equal approximately $100k_B T$. I can conclude that the vortex pinning in the $Nb_{1-x}O_x$ film with small grain structure is close to the limit case of strong disorder but our simple Mendelssohn model describes the vortex creep regime qualitatively only.

4 CONCLUSIONS

The smooth phase coherence appearance in thin films with strong disorder can be explained qualitatively by the increasing of the effective dimensionality of the fluctuation. According to this explanation the smooth transition into the Abrikosov state observed in majority of samples differs qualitatively from the ideal case. Thin films are considered in the present work because the case of bulk superconductors is more difficult. But I think that the transition into the Abrikosov state in bulk superconductors is smoothed out by disorders in consequence of similar cause as considered in this paper. Real superconductors with disorder can be considered as an intermediate case between the Mendelssohn and Abrikosov models. Because intermediate cases are difficult for theoretical description the resistive properties even thin films with disorder can be described qualitatively only.

The second critical field H_{c2} is no critical point not only in superconductors with weak disorder but also in superconductors with strong disorder. The phase coherence appears below H_{c2} in superconductors with weak disorder [3, 4] and above H_{c2} in superconductors with strong disorder.

The sharp transition into the Abrikosov state predicted by the fluctuation theory in ideal case is observed in bulk superconductors with weak disorder [3, 4] only. No sharp transition is observed in thin films with weak disorder [24]. This difference can be explained by difference of the fluctuation value in three- and two-dimensional superconductors.

The mean field approximation can not be used for the description of the mixed state in the thermodynamic limit, because according to [27] the fluctuation value in the Abrikosov state calculated in the linear approximation increases with superconductor size increasing. But according to [27] the mean field approximation can be valid in superconductors with finite sizes: at $h_{c2} - h \gg (\ln(L/\xi))^{2/3} G_{i3D}^{1/3} (th)^{2/3} / 0.16$ in bulk (three-dimensional) superconductor and at $h_{c2} - h \gg (L/0.06\xi)(G_{i2D}th)^{1/2}$ in thin film (two-dimensional superconductor). Here L is sample size across a magnetic field; ξ is coherence length; $G_{i3D} = (k_B T_c / H_c^2(0) \xi^3(0))^2$ is the Ginzburg number of three-dimensional superconductor. For real values $L = 1mm$ and $\xi = 10^{-5}mm$, $\ln(L/\xi) \simeq 10$ whereas $L/\xi = 10^5$. Because for conventional superconductors $G_{i3D} = 10^{-5} - 10^{-11}$ and $G_{i2D} = 10^{-2} - 10^{-6}$ the mean field approximation is not valid in a narrow region near H_{c2} only in bulk superconductors and in whole mixed state of thin films. Therefore the transition into the Abrikosov state in bulk superconductors can be like the ideal transition. Whereas in thin films the phase coherence can

appear in consequence of disorder only.

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Figure Captions

Fig.1. The $(1 + (\Delta\sigma/\sigma_n)(h/t)^{0.5})^{-1}$ versus $(t - t_{c2})/(ht)^{0.5}$ dependencies of the $Nb_{1-x}O_x$ film with small grain structure in magnetic fields $H = 4$ kOe ($h = 0.12$) and $H = 12$ kOe ($h = 0.36$). Film thickness $d = 20$ nm. The measuring current $j = 10^6$ A/m². The current-voltage characteristics are Ohmic in the shown region of temperature and magnetic field values.

Fig.2. The $\Delta\sigma(Gi_{2D}ht)^{0.5}d/t\sigma_0$ versus $(h - h_{c2})/(Gi_{2D}ht)^{0.5}$ dependencies of the $Nb_{1-x}O_x$ film with small grain structure ($T_c = 5.7$ K; $Gi_{2D} \simeq 0.0005$) at $T = 4.2$ K (curve 1) and the amorphous $Nb_{1-x}O_x$ film ($T_c = 2.37$ K; $Gi_{2D} \simeq 0.00015$) at $T = 1.72$ K (curve 2). $t = T/T_c \simeq 0.7$ for both curve. Thickness of both films is equal $d = 20$ nm.

Fig.3. Current-voltage characteristics of the $Nb_{1-x}O_x$ film with small grain structure (with $d = 20$ nm) in magnetic fields $H = 6$ kOe (1); $H = 5$ kOe (2) and $H = 4$ kOe (3) at $T = 4.2$ K ($H_{c2} = 9.0$ kOe). The lines denote the $E = E_0 \sinh(j/j_0)$ dependencies with $E_0 = 0.27$ V/m and $j_0 = 0.90 \cdot 10^8$ A/m² for $H = 6$ kOe; $E_0 = 0.024$ V/m and $j_0 = 0.60 \cdot 10^8$ A/m² for $H = 5$ kOe; $E_0 = 0.0015$ V/m and $j_0 = 0.43 \cdot 10^8$ A/m² for $H = 4$ kOe.

Fig.4. Sketches of the two-dimensional Mendelssohn sponge (A) and of two-dimensional superconductors with disorders (pinning centers) (B) in the Abrikosov state. Superconducting regions are dark. Nonsuperconducting regions are light. Pinning centers are represented by rectangles. Cells with the Abrikosov vortex are marked by 1, without the vortex - 0. Size of the non-superconducting regions b in Mendelssohn sponge is larger than the correlation length (A) whereas the one of pinning centers is smaller than the correlation length (B). The Abrikosov vortexes destroy of superconductivity in circles with radius $r_c > b$ (B). Transport current j increases the rate of thermally activated jumps of vortexes to the right and decreases it to the left (A). w is width of superconducting threads (A).

Fig.5. A) The $(S_j l_j)^{1/3} = (k_B T / j_0 H d)^{1/3}$ versus $(\Phi_0 / H)^{1/2}$ dependence (points with error bars). The line denotes the distance between the Abrikosov vortexes in the triangular lattice. B) The $\ln(E_0 / E_{0,Hc2})$ versus $(H_{c2} / H)^{0.5} - 1$ dependence. $E_{0,Hc2} = 6.9$ V/m; $H_{c2} = 9.0$ kOe. The slope of the line is equal -16. Points with error bars in A) and B) are data obtained from the current-voltage characteristics of the $Nb_{1-x}O_x$ film with small grain structure ($d = 20$ nm) at $T = 4.2$ K (see Fig.3).







